

Method for Reducing the Tangential Velocities in Aircraft Trailing Vortices

Hubert C. Smith*

The Pennsylvania State University, University Park, Penn.

A method is presented to reduce the tangential velocities in the trailing vortex system shed from an airplane wing. The device employed is a porous section of the wing at the tip which essentially tends to equalize the pressure on the upper and lower surfaces. Experiments were conducted on a full-scale aircraft in flight and vorticity measured by probes devised specifically for this purpose. Results showed significant reduction in tangential velocities close behind the wing and to a lesser degree far downstream. Controllable porosity is suggested as a means of reducing the vortex tangential velocities and subsequent potential induced rolling moment imposed on a following aircraft.

Nomenclature

c	= wing chord
c_0	= midspan chord
\bar{c}	= mean chord
c_l	= section lift coefficient
C_L	= wing lift coefficient
r	= radial coordinate
R	= specific radial distance
V	= aircraft forward velocity
V_θ	= tangential velocity of vortex
V_r	= radial velocity of vortex
W	= weight
Z	= downstream distance
Γ	= circulation
Γ_∞	= total circulation
Γ_0	= midspan circulation
ζ	= vorticity
ζ_0	= vorticity at vortex center

Introduction

THE hazard of trailing vortices to following aircraft is well known. The effect is particularly pronounced when large aircraft are operating at low speed such as in terminal area operations, since vortex strength is proportional to aircraft weight and the reciprocal of velocity. A number of devices have been employed in an attempt to attenuate trailing vortex strength, some of which have been moderately successful. This report deals with a somewhat unique method of potential vortex hazard reduction which has not previously been reported in the literature. Even though the research was performed some years ago, the results seem to correlate well with more recent findings and may be of current interest and importance.

This study deals with the employment of a porous wing tip. This is a wing tip with vertical holes allowing some of the higher pressure air on the lower surface to bleed to the lower pressure area on the top surface (see Fig. 1). Previous investigations have shown this method to be successful on wind-tunnel models. However, this is believed to be the only investigation of porous tip effects with a full-scale airplane in flight.

The study is primarily experimental in nature and deals largely with full-scale airplane tests. The airplane used was a

U.S. Army Cessna 0-1A loaned to The Pennsylvania State University for research purposes. Some wind-tunnel studies were also made with a 1/12 scale model of the 0-1 semiwing with porous tip.

The vorticity field close behind the wing in flight was measured by use of the vortex probe developed by May.¹ Additional tests were made with the vortimeter used by Sherrieb² to measure the vortex far downstream. Various characteristics of the vortex are determined from the results and compared with those of the studies of Tangler³ and Sherrieb² for the 0-1 with standard tips.

Previous Investigation

Studies were conducted in 1965 by Tangler³ of the vorticity field behind the 0-1 wing with standard tips. He measured tangential velocities as high as 55% of the freestream velocity. The purpose of his investigation was to investigate the formation of a trailing vortex system and the changes that occur in the vorticity distribution as it moves downstream in the area close behind the wing. These results serve as a source for comparison with similar characteristics of the vortex from the porous tip.

Sherrieb² investigated the tangential velocities of the trailing vortices of several aircraft far downstream by use of an instrument he developed called a vortimeter. One of the aircraft utilized was the 0-1. The experiments were conducted with both standard and porous tips and a brief mention of porous tip results is made in his report. These experiments, conducted jointly by Sherrieb and the author of this report, are further analyzed here.

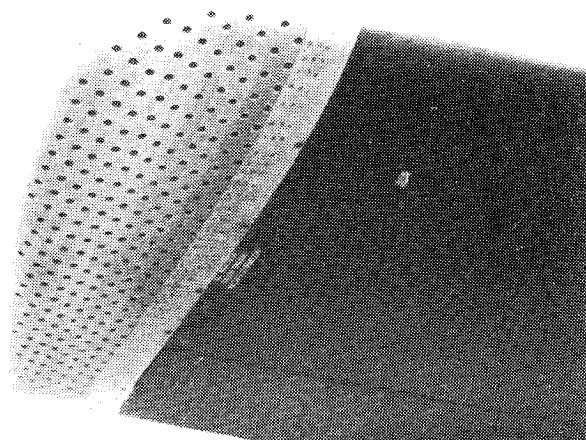


Fig. 1 Porous wing tip on 0-1 aircraft.

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Index categories: Aerodynamics; Jets, Wakes and Viscid-Inviscid Flow Interactions.

*Assistant Professor of Aerospace Engineering. Member AIAA.

The first known investigation of porous tip effects is that of McCormick, Spencer, and Sternfeld.⁴ The purpose of their studies was to investigate various means of reducing tangential velocities in the vortices shed from rotor blade tips. Various devices were applied to a model rotor blade tip in a wind tunnel and the vorticity in the resulting vortex was measured by the vortex probe of May.¹ The porous tip seemed to be one of the most effective methods and one of the simplest to construct. Various degrees of porosity were examined from 10 to 40%. The highlights of all of this research were combined in the paper of McCormick, Tangler, and Sherrieb.⁵

Theoretical Considerations

Vorticity is defined as the curl of the velocity vector, which in two-dimensional polar coordinates can be written as:

$$\zeta = \frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r} - \frac{1}{r} \frac{\partial V_r}{\partial \theta} \quad (1)$$

where

$$\begin{aligned} r &= \text{radius} \\ V_\theta &= \text{tangential velocity} \\ V_r &= \text{radial velocity} \end{aligned}$$

If the flow is axisymmetric (in which case $\partial V_r / \partial \theta = 0$) the equation reduces to

$$\zeta = \frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r} \quad (2)$$

Multiplying by $r dr$ yields

$$\begin{aligned} r \zeta dr &= r dV_\theta + V_\theta dr \\ r \zeta dr &= d(rV_\theta) \end{aligned} \quad (3)$$

Thus, the tangential velocity and vorticity can be related, in integral form, by,

$$V_\theta = \frac{1}{R} \int_0^R \zeta r dr \quad (4)$$

For axisymmetric, two-dimensional flow, the circulation and velocity are related by,

$$\Gamma = 2\pi R V_\theta \quad (5)$$

Therefore, the circulation can be obtained from vorticity measurement as,

$$\Gamma = \int_0^R 2\pi r \zeta dr \quad (6)$$

The lift distribution across a wing varies from a maximum at midspan to zero at the tips. The vortex sheet shed from such a distribution eventually rolls up into a pair of distinct trailing vortices. As the vortex rolls up, the strength of each trailing vortex asymptotically approaches the midspan value of the bound circulation, Γ_0 . Thus,

$$\Gamma_\infty = \Gamma_0 = \frac{1}{2} c_{l0} V \quad (7)$$

The section lift coefficient is related to the wing lift coefficient by a factor which depends upon the wing geometry. Generally, it follows that Γ_0 can be expressed by,

$$\Gamma_0 = \frac{1}{2} c_{l0} (K_1 C_L + K_2) V \quad (8)$$

The constants K_1 and K_2 can be determined by calculation of the lift distribution by methods such as that of Glauert⁶ or Shrenk.⁷ Determination of these constants for the 0-1 wing results in a midspan circulation given as,

$$\Gamma_0 = 2.76 (1.08 C_L + 0.07) V \quad (9)$$

Experimental Investigation

Porous wing tips were designed and constructed for the Cessna 0-1. A porosity of 10% was chosen as a result of studies by McCormick, et al.⁴ These studies showed that 10% porosity significantly reduced the maximum tangential velocities shed from a porous tip rotor blade but did not appreciably increase profile drag. Each tip was increased 5.7 in. in span in order to give a span of porous area equal to 30% of the tip chord. This again was to approach the configuration of Ref. 4. A span of 35% of chord was used there and gave satisfactory results.

The tips were constructed of solid balsa wood planking laminated together, shaped, and then coated with fiberglass-reinforced plastic. Two-hundred-fifty-seven equally spaced holes of 1/2 in. diameter were drilled through the entire assembly, which gave an actual porosity of 11.3% (10% nominal). The fiberglass coating was extended inboard 1 in. to form a flange by which the tips were attached to the end rib of the wing similar to attachment of the standard tip. Both tips were constructed identically so as not to disturb the symmetry of configuration and hence the roll characteristics of the airplane. The installed porous tip is shown in Fig. 1.

The airplane was then fitted with the same equipment used by Tangler for his studies of the vortex system with standard tips. This equipment consists basically of a framework which supports a mechanism to position a vortex probe at various locations behind the wing. This apparatus is shown in Fig. 2. The probe supporting mechanism can be positioned vertically in flight by a motor driving a worm gear. The entire vertical traversing mechanism can be moved inboard or outboard and fore and aft by manual adjustment on the ground. Transverse planes were surveyed at locations 1, 2, 4, and 6 ft behind the trailing edge of the wing. The traversing mechanism was placed at every inch spanwise through the main vortex and vertical readings taken about every 1/4 in. through the center of the vortex.

The probe itself consists of a set of four unpitched vanes connected to a shaft through which a hole is drilled perpendicular to the axis. On one side of the hole is a small light bulb and on the other is a photocell. The photocell is connected to an electronic counter which measures the rotational velocity from the impulses of the photocell. Since it gives two impulses for every rotation due to such a configuration, the actual rotational velocity is one-half of that recorded. The rotational velocity recorded, divided by two, is therefore considered to be the rotational velocity of the fluid at that particular location.



Fig. 2 Test aircraft in flight with probe mount installed.

The complete trailing vortex system was surveyed in four transverse planes for each of two velocities, 65 and 75 knots indicated airspeed. Flights were made at an average pressure altitude of 6500 ft. The weight was taken as an average operating weight. This average weight included the empty aircraft plus equipment, two crew members, and half fuel load, which was calculated as 2136 lb. The porous tips added an additional 3.5 ft² of wing area to give a total of 177.5 ft². Using these values the lift coefficients for the two airspeeds flown were calculated as 0.85 for 65 knots and 0.63 for 75 knots indicated airspeed.

Tests were also conducted to determine the tangential velocities from the porous tip far downstream by use of the vortimeter developed by Sherrieb. This instrument consists of an array of 1/4 in. diameter cylinders, the drag of which can be measured through attached strain gages connected to a recording oscillograph. This instrument was placed with the cylinders parallel to the runway and the aircraft was flown by at various distances from the vortimeter. The tangential velocities (in terms of drag) were recorded as the vortex sweeps across the cylinders. By measuring the time from the passing of the aircraft until the vortex was recorded, the distance of the vortex from the aircraft could be determined, knowing the speed of the aircraft.

In addition to the flight tests, wind-tunnel tests were conducted in a low-speed tunnel. A 1/12 scale model

semiwing of the Cessna 0-1 was used with a 10% porous tip attached. The vortex probe similar to that used for full-scale flight tests was again utilized except that smaller vanes of 3/8 in. diameter were fitted to it.

Results and Analysis

Contour plots of constant rotational velocity in terms of rpm were plotted as shown in Figs. 3 and 4 for the in-flight measurements. Compared with Tangler's plots of similar data for standard tips, these show a general trend of greatly reduced rotational velocities. The maximum measured rotational velocities for 65 knots are 21 to 24% of the maximum velocities for similar positions with standard tips. For 75 knots, the maximum velocities are 45 to 74% of similar values for standard tips. Contours for similar rpm values do not appear to be spread out very much more for porous tips than for standard tips. The most noticeable quality is the lack of high rotational velocities in the center of the porous tip vortices.

Another interesting result is the existence of two centers in the vortices of certain transverse planes. This is particularly noticeable in the 4 ft aft position in Fig. 4. All of the vortex contours, however, indicate a flattened-out shape which could possibly contain two centers. It is quite possible that the probe was not positioned exactly in the second center. The measurement is very critical near the center, where the vorticity gradient is high. This double vortex might well result from one forming at the end of the solid wing (beginning of the porous area) and another at the actual tip. Wind-tunnel tests with the 1/12 scale model semiwing showed this same result. The contour plots of the model wing data also bear out the trend of reduced rotational velocities when compared with Tangler's results for the standard tip wing model.

The vortex sheet did not appear to be significantly different from that of the standard tips, except in one respect. That is, it seemed to extend much farther inboard. This is true for both the full-scale airplane and the model. It should also be pointed out that readings in the vortex sheet were difficult to obtain, particularly in flight, since the velocities were of the same order as small perturbations in the atmosphere and those resulting from slight movements of the aircraft. For this reason, the accuracy of data obtained on the inboard vortex sheet is questionable.

The circulation for the resulting vortex was determined as follows. Using Eq. (6) for a circular vortex,

$$\Gamma = \int_0^{\infty} 2\pi r \zeta dr \quad (10)$$

and substituting twice the angular velocity (measured in rpm) for vorticity,

$$\Gamma = \frac{2\pi}{30} \int_0^{\infty} \text{rpm} (2\pi r) dr \quad (11)$$

A numerical approximation to this equation can be made in the form

$$\Gamma = \frac{2\pi}{30} \sum_0^{\infty} \text{rpm} (A) \quad (12)$$

where A is the area within the contour of constant rpm. Thus, by graphically integrating the areas within contours of constant rpm and summing them, an approximate Γ can be determined even for vortices of other than exact circular shape. This was done for both the rolled-up vortex and the vorticity in the vortex sheet. Note that the meter has a certain limit to its sensitivity. A low vorticity spread out over a large area could result in a significant Γ yet not be detected by this

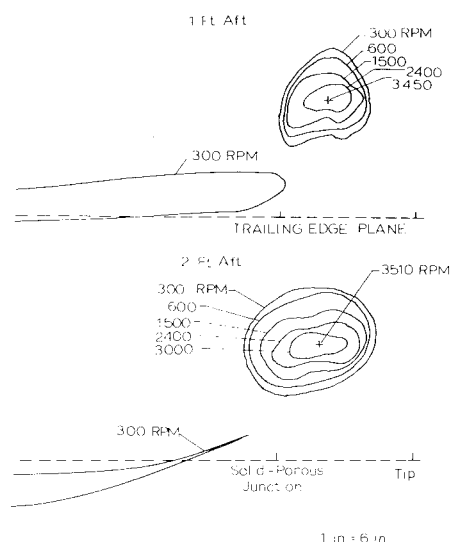


Fig. 3 Contours of constant rotational velocity at 65 knots.

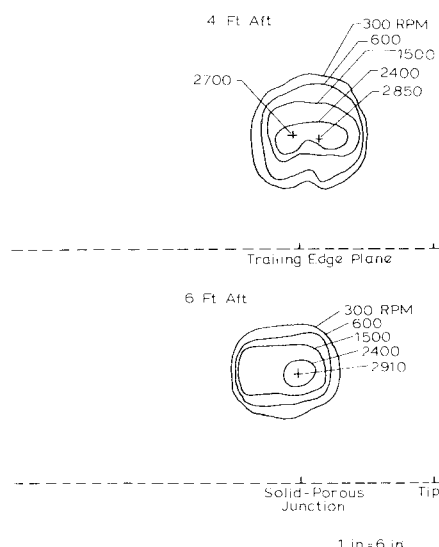


Fig. 4 Contours of constant rotational velocity at 65 knots.

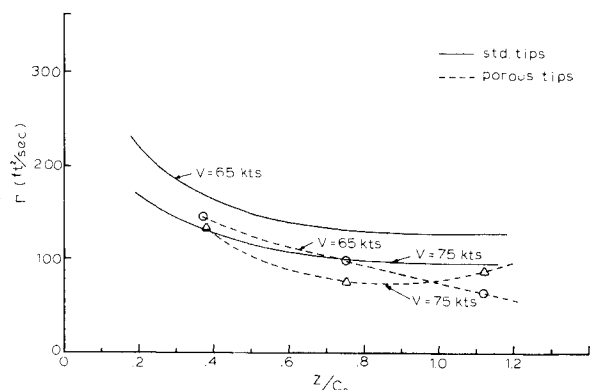


Fig. 5 Total circulation vs downstream distance.

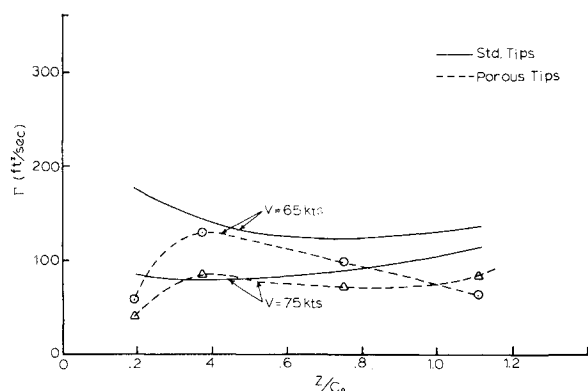


Fig. 6 Circulation in rolled-up portion of vortex vs downstream distance.

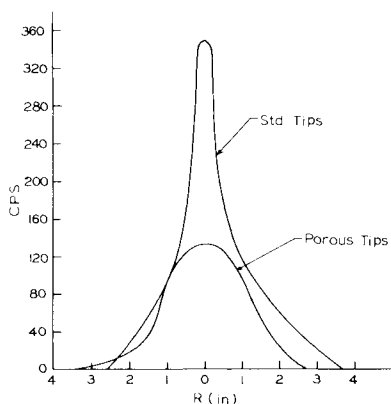


Fig. 7 Comparison of vorticity distribution at 1 ft aft and 75 knots velocity.

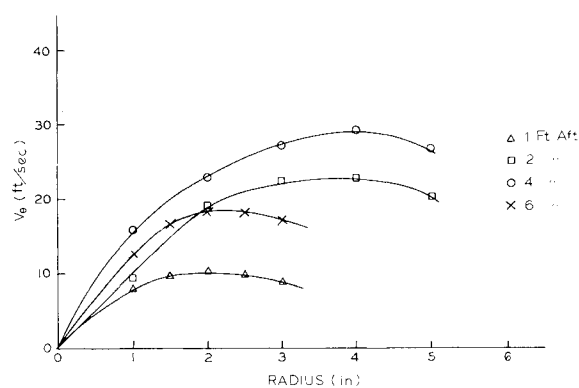


Fig. 8 Tangential velocity vs radius of vortex at 65 knots.

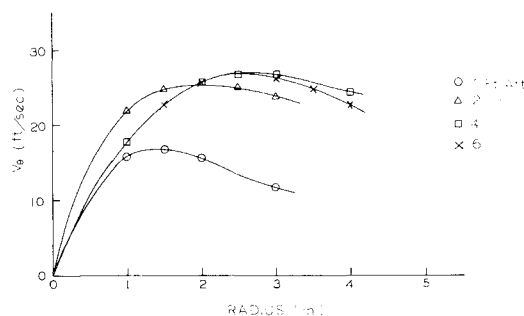


Fig. 9 Tangential velocity vs radius of vortex at 75 knots.

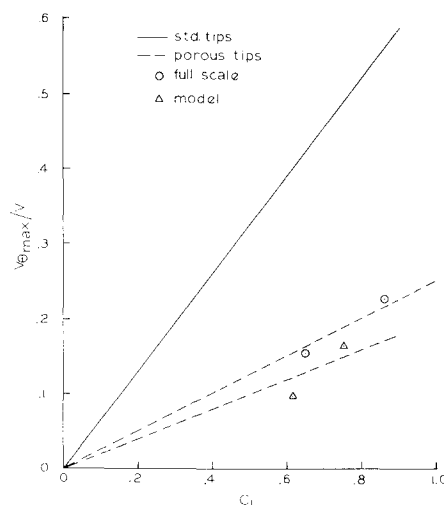


Fig. 10 Nondimensional maximum tangential velocity vs lift coefficient.

method. However, the objective was to determine the relative strength of the vortex from the porous tip as compared to that of the standard tip. For this purpose the method seems valid, even though exact values of Γ may not be determined.

The total Γ for in-flight measurements vs nondimensional distance downstream is presented in Fig. 5 and compared with standard tip results. Similar data are presented in Fig. 6 for the rolled-up portion of the vortex only. These curves show circulation close to that of the standard tip but generally slightly lower for both airspeeds. The first station measured (1 ft aft) however, is lower by 33% for 65 knots and 48% for 75 knots. This is probably reasonable for the rolled-up vortex since much of the circulation is shed inboard in the sheet. It is assumed that a lack of additional circulation in the sheet is due to failure to measure it accurately. Therefore, the low values for 1 ft aft are considered in error and not included in the curve in Fig. 5.

Changes in vortex structure are emphasized by measurement of the vorticity. Figure 7 shows a typical vorticity distribution measured at the same position behind the wing for both the standard and porous tips. This illustrates the greatly reduced vorticity at the center of the porous tip vortex. The contour is more spread out, however, so that the overall size of the vortex is about the same as that from the standard tip. To obtain the tangential velocity V_θ as a function of radius, a curve of values of ζr vs r was plotted and graphically integrated using Eq. (4).

V_θ vs radius is then plotted in Figs. 8 and 9 for various stations aft. $V_{\theta \max}$ is determined by taking an average of the maximum values of the four curves. $V_{\theta \max}/V$ is then plotted against C_L on Fig. 10 and compared with standard tip data. Some corrections were applied to Tangler's data for in-flight C_L values which shifted his curve for in-flight data so that it coincided almost exactly with that of his model tests. Hence,

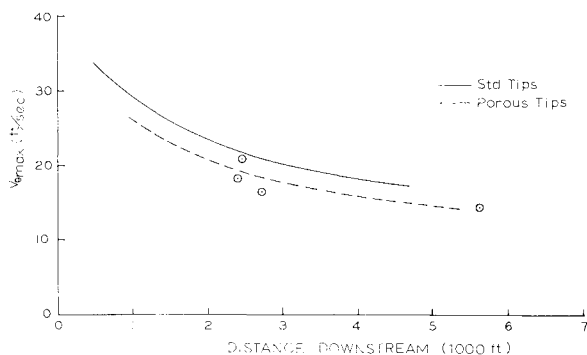


Fig. 11 Maximum tangential velocity vs downstream distance.

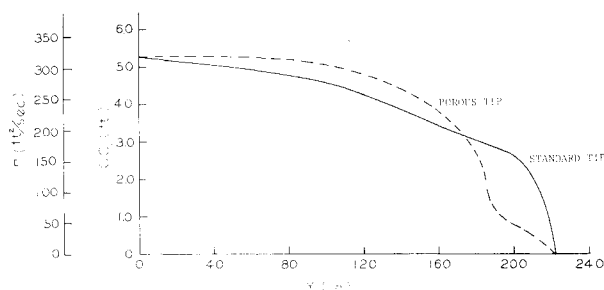


Fig. 12 Lift distribution on 0-1 wing with and without porous tip.

only one curve appears here for standard tip data which represents both model and full-scale results. $V_{\theta_{max}}$ for model porous tip tests is determined also by Eq. (4). On this graph can be seen the most significant results of these tests. That is, that the maximum tangential velocities are greatly reduced by use of the porous tips. The full-scale results show the tangential velocities to be reduced by about 60% and the model tests almost 70%.

This picture is not quite so optimistic, however, when the results of measurements far downstream are considered as presented in Fig. 11. Here, the reduction appears to be on the order of 10% of those velocities measured with standard tips. The overall results seem to imply that the porous tips shift the circulation inboard and thereby greatly reduce the strength of the tip vortex close behind the wing. The vortex sheet, which contains much of the circulation, then eventually rolls up into the tip vortex. However, since this appears to be delayed by the porous effect, some of its strength is lost through dissipation in the atmosphere. When it finally rolls up it appears to have only about 90% of the strength of the standard tip vortex.

The in-flight results of $V_{\theta_{max}}$ as shown on Figs. 8 and 9 are not sufficiently conclusive to predict a trend of increasing or decreasing $V_{\theta_{max}}$ as the vortex progresses downstream. However, if one neglects the results of the 6 ft position in the 65 knots case, which is subject to more error than the other positions, an increasing trend in $V_{\theta_{max}}$ is shown with increasing Z . This is true for both 65 and 75 knot results. This also points toward a possibility of the vortex building up beyond 1 chord length downstream. It is assumed, then, that a complete rolling up occurs somewhere between 1 and 400 chord lengths downstream. However, with the available equipment, no means could be devised to measure the vortex adequately in this region. Hence, one must resort to speculating on the behavior in this area from results obtained farther upstream and farther downstream.

Comparison with Subsequent Research

The reduction in the circulation of the rolled-up vortex measured immediately behind the wing with the porous tip over that of the standard tip suggests an inboard shift of the

spanwise lift distribution such as is shown in Fig. 12. The shift in span loading is further interpreted as having an effect on the reduction of tangential velocity which is measured when roll-up is completed. This fact correlates well with subsequent studies conducted primarily to determine the effect of flaps on trailing vortex strength.

Rossow⁸ predicted that unloading of the outboard portion of the wing by inboard flaps would reduce maximum tangential velocities in the vortex wake. This prediction was borne out by experimental studies conducted in the NASA Ames 7×10 ft wind tunnel by Orloff and Ciffone⁹ for measurements close behind the wing. Additional studies by Ciffone and Orloff¹⁰ showed that this effect was applicable to distances as great as 100 span lengths downstream. Corsiglia and Dunham¹¹ further supported these findings with both ground-based and flight tests in which they measured rolling moments induced in vortex wakes with and without flaps.

Another significant finding with the porous tips was the existence of two separate vortices on each wing as shown in Fig. 4. One of these is presumed to be generated by the junction of the porous tip and solid portion of the wing. Rossow¹² showed that vortices rotating in opposite directions as shed from flap edges or other wing devices will interact and tend to reduce wake velocities. Iverson, et al.,¹³ further showed that corotational vortices would have similar effects upon merging. Their wind-tunnel measurements with hot-wire anemometers showed that peak tangential velocities from two merged vortices were no higher than those from the individual vortices. Croom¹⁴ found that induced rolling moments were reduced behind wings with mid-semispan spoilers deployed, which, again, points to multiple vortex formation. Faery and Marchman¹⁵ also showed that winglets produce two distinct vortices at the wing tip, each of which have about 65% less swirl velocity than that produced by simple rounded tip configuration.

El-Ramly and Rainbird¹⁶ evaluated a number of different vortex alleviation devices and concluded that no single device could significantly reduce the trailing vortex hazard. However, with a combination of various methods, induced rolling moments could be reduced by as much as 25%. Morris¹⁷ went on to conclude that incorporation of many of these devices is operationally feasible on heavy commercial jet transports. Those in which adverse effects can be minimized are, of course, the most desirable. In light of these findings, it is considered that the porous wingtip, while not in itself, a drastic vortex attenuator, could be utilized in conjunction with other methods to yield significant vortex hazard reduction.

Conclusions and Recommendations

Experiments have been conducted in flight to measure the vorticity field close behind a wing fitted with a porous tip. Additional tests were performed to measure the strength of the vortex from this tip several thousand feet downstream. Comparing the results with those previously obtained from the same wing in standard configuration leads to the following conclusions:

- 1) The porous wingtip is an effective method of reducing tangential velocities in the trailing vortex close behind the wing. The 10% porous tip shows reductions up to 60% in the vicinity of 1 chord length behind the wing.
- 2) The porous wing tip is moderately effective in reducing tangential velocities far downstream. The 10% porous tip yields reductions of about 10% several thousand feet downstream.
- 3) The porous wing tip shifts the spanwise lift distribution inboard so that more of the circulation is shed inboard.
- 4) Two distinct vortices are shed from the porous wing tip, one at the actual tip and one at the junction of the porous tip and solid portion of the wing.
- 5) A delay in the rolling up of the vortex sheet into the trailing vortex is introduced by the porous wing tip. The

complete rolling up occurs somewhere between 1 and 400 chord lengths downstream.

No detailed studies were conducted to determine the amount of drag generated by the porous tips. It is assumed that profile drag would be increased by the porous characteristics. However, the nature of the porous tip lends itself very well to the design of a device which would allow the opening and closing of the porous shafts in flight. In this way any increase in drag could be eliminated, for example, during cruise flight. During operations such as landing approach, the shafts could be opened. It is in such situations when the vortex hazard is most critical and additional drag could be tolerated, if not even desired.

Thus, it appears that porous wing tips do have a definite, although slight, effect on the reduction of tangential velocities in trailing vortices. However, this effect may be significant when used in conjunction with other devices which also show similar effects. Its potential for minimal adverse effects on airplane performance make it a particularly desirable means of vortex attenuation. It is recommended that further study be conducted to determine the effect of the porous wing tip in combination with other vortex hazard alleviation methods.

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